International Journal of Engineering & Scientific Research Vol.4 Issue 12, December 2016, ISSN: 2347-6532 Impact Factor: 5.900 Journal Homepage: <u>http://www.ijmra.us</u>, Email: editorijmie@gmail.com Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

AUTOMATIC DETERMINATION OF PID CONTROLLER PARAMETERS BY USING SIMULINK AND ITS JUSTIFICATION IN SIMULATING VEHICLE ACTIVE SUSPENSION SYSTEM

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Abstract

This paper will present the development of oscillatory models of active suspension system. It is accomplished by modifying oscillatory models of passive suspension system by introducing active force into the system presented by PID controller. One-dimensional, two-dimensional and three-dimensional vehicle oscillatory models with active suspension system are formed. Adjustment of PID controller parameters was conducted automatically by using Simulink programme. Analysis and comparison of individual oscillatory parameters for one-dimensional, two-dimensional and three-dimensional oscillatory model of passive and active suspension system were also completed. The obtained results of vertical movement of sprung mass (vehicle body) and unsprung mass (wheels), as well as sprung mass angle rotation around x and y axes of a vehicle justify the use of PID controller for simulating the behaviour of a vehicle active suspension system.

Key words:PID controller, Simulink, active suspension, vertical oscillations, sprung mass, unsprung mass

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1. Introduction

Movement of a vehicle on the road is always accompanied by the influence of external and internal forces, which are transferred to the vehicle through wheels. As the vehicle bodyleans on the wheels by means of suspension system, this causes the occurrence of oscillation of the vehicle body in direction of the vertical axis of the vehicle and around the axis of the vehicle. Suspension system is an important system whose role isto provide comfort and safety to passengers and cargoes for all road vehicles. These two parameters are always in negative conjunction with each other, which means that a compromise between safety in the movement of vehicles and passenger comfort must be created. It is possible to influence these two elements through damping characteristics andstiffness elements of suspension system for classic (passive) suspension systems. [1, 2].

However, during vehicle movement and particularly under the influence of inertial forces in the longitudinal motion, as well as the effects of centrifugal force of a vehicle in a curve, significant oscillations of the vehicle sprung mass occur. The aforementioned swinging of a sprung massas a result of theinfluence of the gyroscopic moment additionally undermines stability of a vehicle.

Due to this, recently an active research has been developed on the introduction of semi-active and active suspension for modern passenger vehicles, which aim to minimize or even completely eliminate the swinging of asprung mass. In order to achieve this task, certain elements in suspension system must have variable parameters of damping and stiffness [3]. Oscillatory vehicle models are introduced to conduct research on oscillatory behaviour of vehicles in the analysis of performance of suspension systems [4]. Oscillatory models can be solved and analysedby using any of the MBS (Multi Body Simulation) programmes, or numerically by using MATLAB or by simulation using Simulink. One of the most commonly used software for analysis of these models is Simulink within package programme Matlab [5]. Modellingof active suspension system in Simulink is normally reduced to optimization programme of this controller through analysis of ¹/₄ vehicle model [6]. The general equation of this controller is given by equation (1), and the general scheme of the controller is shown in Figure 1 [7].

$$x(t) = K_p \left[u(t) + \frac{1}{T_i} \int_0^t u(t) + T_d \frac{du}{dt} \right]$$
(1)

The most important segment in optimizing PID controller is determining its parameters Kp, Ki and Kd. In the example shown in [8] Ziegler-Nichols method, which is used to determine these parameters, was explained. However, Simulink provides the possibility of automatically determiningparameters and in this paper these parameters will beautomatically determined using the block of PID controller in Simulink programme. Some parameters of oscillatory behavior of the sprung mass (body) and the unsprung mass (wheel) in the case of passive and active suspension system will be analysed and compared. The analysis will be carried out for cases ¹/₄, ¹/₂ and a full vehicle model.



Figure 1. Block structure of PID controller

2. Forces and moments acting on a vehicle

The equations of vehicle motion in the study of vehicle dynamics are usually represented by a system of equations in the coordinate system (Txyz) related to the vehicle in the center of gravity of the vehicle T. The *x*-axis is longitudinal and it is directed at the vehicle, the *y*-axis isperpendicular to the lateral vehicle plane, and the z-axis is perpendicular to the horizontal vehicle plane. The resultant forces and moments are shown in Figure 2. in the directions x, y and z-axes which can act on a vehicle in general case of vehicle movement.

The general equations for resulting force and moment which act on a vehicle are presented by the following equation:

$$F = F_x \vec{\iota} + F_y \vec{j} + F_z \vec{k}$$

$$M = M_x \vec{\iota} + M_y \vec{j} + M_z \vec{k}$$
(2)

Whereas

- F_x resulting force acting on a vehicle in the direction of x-axis,
- F_y resulting force acting on a vehicle in the direction of y-axis,
- F_z resulting force acting on a vehicle in the direction of z-axis,
- M_x resulting moment acting on a vehicle around x-axis,
- M_y resulting moment acting on a vehicle around y-axis,
- M_z resulting moment acting on a vehicle around z-axis.

In order to form the oscillatory model, the effect of all the forces is neglected, apart from the vertical one. Next, the oscillatory models of vehicles for the analysis of activesuspension systems are presented.



Figure 2. Vehicle with coordinate system

3. Vehicle oscillatory models

3.1. One-dimensional vehicle model

The most commonly used model of vehicles during the analysis of oscillatory behaviour on vehicles, especially passenger vehicles, is one-dimensional model with two masses (sprung mass m_V and unsprung mass m_T), as shown in Figure 3. This model is often called ¹/₄ of vehicle model.

Figure 3.shows that the influence of damping in the tire is ignored for this analysis.



Figure 3. One-dimensional oscillatory model with two masses

Equations of oscillation of sprung mass – the vehicle body (m_V) and unsprung mass - wheels (m_T) in the direction of *z*-axis is presented next. Equation of oscillation of sprung mass in the *z*-axisdirection is as follows:

$$m_{V} \cdot \frac{d^{2} z_{V}}{dt^{2}} + k_{V} \left(\frac{d z_{V}}{dt} - \frac{d z_{T}}{dt}\right) + c_{V} \left(z_{V} - z_{T}\right) - f = 0$$
(3)

Equation of oscillation of unsprung mass in *z*-axis direction is as follows:

$$m_T \cdot \frac{d^2 z_T}{dt^2} + k_V \left(\frac{dz_T}{dt} - \frac{dz_V}{dt}\right) + c_V (z_T - z_V) + c_T (z_T - z_P) + f = 0$$
(4)

In the equations (3) and (4) parameter *f* presents the force which is used by active suspension system to perform settling and damping of oscillations of sprung mass of vehicles (m_V) .

3.2. Two-dimensional vehicle model

This model is also called the model of half of a vehicle. Two-dimensional model replaces the vehicle more precisely because it takes into consideration suspension of sprung mass over two or more axles. Based on this model it is possible to analyse vertical as well as oscillations around the axis perpendicular to longitudinal plane of the analysed vehicle oscillatory model (m_V). Model which replaces the vehicle during the analysis of oscillatory behaviour of a vehicle is shown in Figure 4.

Figure 4. shows sprung mass (m_V) , front axle unsprung mass (m_{TP}) , rear axle unsprung mass (m_{TZ}) . It can also be seen that sprung mass has an inertion moment around y-axis (axis

perpendicular to longitudinal plane) marked as J_Y . Active forces which perform settling down of vertical oscillations are marked as f_P and f_Z .



Figure 4. Two-dimensional oscillatory model with three masses

Equations which describe the given model are based on the conditions of dynamic balance presented with the following equations:

– Equation (5) illustrates the sum of all forces acting on sprung mass in *z*-axis direction:

$$m_{V} \frac{d^{2} z_{V}}{dt^{2}} + k_{VP} \left(\frac{d z_{V}}{dt} - \frac{d z_{TP}}{dt} - a_{1} \frac{d \theta}{dt} \right) + k_{VZ} \left(\frac{d z_{V}}{dt} - \frac{d z_{TZ}}{dt} + a_{2} \frac{d \theta}{dt} \right) + c_{VP} \left(z_{V} - z_{TP} - a_{1} \theta \right) + c_{VZ} \left(z_{V} - z_{TZ} + a_{2} \theta \right) - f_{P} - f_{Z} = 0$$
(5)

– Equation (6) illustrates the sum of all moments around *y*-axis:

$$J_{Y} \frac{d^{2}\theta}{dt^{2}} + a_{1} \cdot k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TP}}{dt} - a_{1} \frac{d\theta}{dt} \right) + a_{2} \cdot k_{VZ} \left(\frac{dz_{V}}{dt} - \frac{dz_{TZ}}{dt} + a_{2} \frac{d\theta}{dt} \right) - a_{1} \cdot c_{VP} \left(z_{V} - z_{TP} - a_{1} \theta \right) + c_{VZ} \left(z_{V} - z_{TZ} + a_{2} \theta \right) + f_{P} a_{1} - f_{Z} a_{2} = 0$$
(6)

– Equation (7) illustrates the sum of all forces acting on front unsprung mass in z-axis direction:

$$m_{TP} \frac{d^2 z_{TP}}{dt^2} - k_{VP} \left(\frac{dz_V}{dt} - \frac{dz_{TP}}{dt} - a_1 \frac{d\theta}{dt} \right) + c_{TP} (z_{TP} - z_{PP}) - c_{VP} (z_V - z_{TP} - a_1\theta) + f_P = 0$$
(7)

– Equation (8) illustrates the sum of all forces acting on the rear unsprung mass in *z*-axis direction:

$$m_{TZ} \frac{d^2 z_{TZ}}{dt^2} - k_{VZ} \left(\frac{dz_V}{dt} - \frac{dz_{TZ}}{dt} + a_2 \frac{d\theta}{dt} \right) + c_{TZ} (z_{TZ} - z_{PZ}) - c_{VZ} (z_V - z_{TZ} - a_2\theta) + f_Z = 0$$
(8)

3.3. Three-dimensional vehicle model

This model is also called the full vehicle model and it has seven degrees of freedom of movement. Seven degrees of freedom are represented by the vertical movement of sprung mass (m_V) and four unsprung masses $(m_{TP} \text{ and } m_{TZ})$ in z-axis direction, as well as angular movements of vehicle aroundx y axis, all of which describe this system. Three-dimensional vehicle model with five masses is presented in Figure 5.



Figure 5. Three-dimensional oscillatory model with five masses

Equations which describe model presented in Figure 5, which were formed from the dynamic balance conditions, are as follows:

– Equation (9) illustrates the sum of all forces acting on sprung mass in *z*-axis direction:

$$m_{V} \frac{d^{2} z_{V}}{dt^{2}} + k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TPL}}{dt} + b_{1} \frac{d\varphi}{dt} - a_{1} \frac{d\theta}{dt} \right) + k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TPD}}{dt} - b_{2} \frac{d\varphi}{dt} - a_{1} \frac{d\theta}{dt} \right) + k_{VZ} \left(\frac{dz_{V}}{dt} - \frac{dz_{TZD}}{dt} - b_{2} \frac{d\varphi}{dt} \right) + k_{VZ} \left(\frac{dz_{V}}{dt} - \frac{dz_{TZL}}{dt} + b_{1} \frac{d\varphi}{dt} + a_{2} \frac{d\theta}{dt} \right) + c_{VP} (z_{V} - z_{TPL} + b_{1}\varphi - a_{1}\theta) + c_{VP} (z_{V} - z_{TPD} - b_{2}\varphi - a_{1}\theta) + k_{VZ} (z_{V} - z_{TZD} - b_{2}\varphi + a_{2}\theta) + c_{VZ} (z_{V} - z_{TZL} + b_{1}\varphi + a_{2}\theta) - - f_{PL} - f_{PD} - f_{ZL} - f_{ZD} = 0$$

$$(9)$$

– Equation (10) illustrates the sum of all moments around *x*-axis:

$$J_{X} \frac{d^{2}\varphi}{dt^{2}} + b_{1}k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TPL}}{dt} + b_{1}\frac{d\varphi}{dt} - a_{1}\frac{d\theta}{dt}\right) - b_{2}k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TPD}}{dt} - b_{2}\frac{d\varphi}{dt} - a_{1}\frac{d\theta}{dt}\right) - b_{2}k_{VZ} \left(\frac{dz_{V}}{dt} - \frac{dz_{TZD}}{dt} - b_{2}\frac{d\varphi}{dt} + a_{2}\frac{d\theta}{dt}\right) + b_{1}k_{VZ} \left(\frac{dz_{V}}{dt} - \frac{dz_{TZL}}{dt} + b_{1}\frac{d\varphi}{dt} + a_{2}\frac{d\theta}{dt}\right) + b_{1}c_{VP}(z_{V} - z_{TPL} + b_{1}\varphi - a_{1}\theta) - b_{2}c_{VP}(z_{V} - z_{TPD} - b_{2}\varphi - a_{1}\theta) - b_{2}c_{VZ}(z_{V} - z_{TZD} - b_{2}\varphi + a_{2}\theta) + b_{1}c_{VZ}(z_{V} - z_{TZL} + b_{1}\varphi + a_{2}\theta) + c_{TS}\varphi + b_{1}(f_{PL} + f_{ZL}) - b_{2}(f_{PD} + f_{ZD}) = 0$$
(10)

– Equation (11) illustrates the sum of all moments around *y*-axis:

$$J_{y} \frac{d^{2}\theta}{dt^{2}} - a_{1}k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TPL}}{dt} + b_{1} \frac{d\varphi}{dt} - a_{1} \frac{d\theta}{dt} \right) - a_{1}k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TPD}}{dt} - b_{2} \frac{d\varphi}{dt} - a_{1} \frac{d\theta}{dt} \right) + a_{2}k_{VZ} \left(\frac{dz_{V}}{dt} - \frac{dz_{TZD}}{dt} - b_{2} \frac{d\varphi}{dt} + a_{2} \frac{d\theta}{dt} \right) + a_{2}k_{VZ} \left(\frac{dz_{V}}{dt} - \frac{dz_{TZL}}{dt} + b_{1} \frac{d\varphi}{dt} + a_{2} \frac{d\theta}{dt} \right) + a_{1}c_{VP}(z_{V} - z_{TPL} + b_{1}\varphi - a_{1}\theta) - a_{1}c_{VP}(z_{V} - z_{TPD} - b_{2}\varphi - a_{1}\theta) + a_{2}c_{VZ}(z_{V} - z_{TZD} - b_{2}\varphi + a_{2}\theta) + a_{2}c_{VZ}(z_{V} - z_{TZL} + b_{1}\varphi + a_{2}\theta) + a_{2}(f_{ZL} + f_{ZD}) - a_{1}(f_{PL} + f_{PD}) = 0$$

$$(11)$$

- Equation (12) illustrates the sum of all forces acting on front left unsprung mass in *z*-axis direction:

$$m_{TP} \frac{d^{2} z_{TPL}}{dt^{2}} - k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TPL}}{dt} + b_{1} \frac{d\varphi}{dt} - a_{1} \frac{d\theta}{dt} \right) - c_{VP} \left(z_{V} - z_{TPL} + b_{1} \varphi - a_{1} \theta \right) - c_{TS} \frac{\varphi}{b_{1} + b_{2}} + c_{TP} \left(z_{TPL} - z_{PPL} \right) + f_{PL} = 0$$
(12)

- Equation (13) illustrates the sum of all forces acting on front right unsprung mass in *z*-axis direction:

$$m_{TP} \frac{d^{2} z_{TPD}}{dt^{2}} - k_{VP} \left(\frac{dz_{V}}{dt} - \frac{dz_{TPD}}{dt} - b_{2} \frac{d\varphi}{dt} - a_{1} \frac{d\theta}{dt} \right) - c_{VP} (z_{V} - z_{TPD} - b_{2} \varphi - a_{1} \theta) + c_{TS} \frac{\varphi}{b_{1} + b_{2}} + c_{TP} (z_{TPD} - z_{PPD}) + f_{PD} = 0$$
(13)

- Equation (14) illustrates the sum of all forces acting on rear right unsprung mass in *z*-axis direction:

$$m_{TZ} \frac{d^2 z_{TZD}}{dt^2} - k_{VZ} \left(\frac{dz_V}{dt} - \frac{dz_{TZD}}{dt} - b_2 \frac{d\varphi}{dt} - a_2 \frac{d\theta}{dt} \right) - c_{VZ} (z_V - z_{TZD} - b_2 \varphi + a_2 \theta) + c_{TZ} (z_{TZD} - z_{PZD}) + f_{ZD} = 0$$
(14)

- Equation (15) illustrates the sum of all forces acting on rear left unsprung mass in *z*-axis direction:

$$m_{TZ} \frac{d^2 z_{TZL}}{dt^2} - k_{VZ} \left(\frac{dz_V}{dt} - \frac{dz_{TZL}}{dt} + b_1 \frac{d\varphi}{dt} + a_2 \frac{d\theta}{dt} \right) - c_{VZ} (z_V - z_{TZL} + b_1 \varphi + a_2 \theta) + c_{TZ} (z_{TZL} - z_{PZL}) + f_{ZL} = 0$$
(15)

4. Setting PID regulator coefficients

There are several methods for adjusting the coefficients of the PID controller, which can be seen in [9]. However, when the coefficients of the PID controller are adjusted in Simulink, it is possible to perform automatic adjustments. Figure 6. shows this way of setting the PID controller in Simulink.

PID Controller					
This block implements anti-windup, external (requires Simulink Cor	continuous- and disc reset, and signal track ntrol Design).	rete-time PID co king. You can tun	ntrol alg ie the PI	orithms and includes D gains automatically	advanced features such using the 'Tune' buttor
controller: PID		•	Form:	Parallel	
Time domain:				Server and the server	
Continuous-time					
Discrete-time					
Main DTD 4.1					
Controller narameter	u Data Types S	tate Attributes	2		
Sources	lintornal			-	E Compensator formu
Source: Internal				•	esinpensator forma
Proportional (P):	1				
Integral (I): 0 Derivative (D): 0					$P + I^{1} + D^{N}$
					$1 + 1 - \frac{1}{s} + D - \frac{1}{1 + N^{-1}}$
Filter coefficient (N): 2152.88594883722					8
				Tune	
Initial conditions					
(C)		111			

Figure 6.Adjustment of PID controller parameters

At the very beginning, it is necessary to enter arbitrary values for the parameters (P), (I) and (D). After that, the button *Tune* is clicked, and the form like the one in Figure 7. is shown.

Figure 7.shows the appearance of a signal to be adjusted, with default values from Figure 6. (dashed line) and the appearance of the signal after the adjustment. The values of the parameters (P), (I) and (D) can be seen in the lower right corner. This appearance of the signal is automatically offered to the programme user who can change its appearance. This is achieved by moving the slider *Response Time* and *Transent Behavior*.



Figure 7. The form for optimization of PID controller parameters

Once the satisfactory values of controller parameters are obtained, the button *Update Block* should be clicked, and then the new parameters are automatically entered into the appropriate fields, as seen in Figure 8.After which the Simulink model can be started and further analysis of the problem can be continued.

PID Con	troller					
This blo anti-win (require	ck implements dup, external r s Simulink Con	continuous- and c eset, and signal tr trol Design).	liscrete-time PID co acking. You can tur	ntrol alg ne the PI	orithms and includes D gains automaticall	advanced features such a γ using the 'Tune' button
ontroller	: PID	-		Form:	Parallel	
Time do Cont Discr	main: inuous-time ete-time					
Main	PID Advanced	Data Types	State Attributes]		
Control	ler parameters					
Source: internal		20		•	Compensator formula	
Proportional (P): 1.89575307082986		50				
Integral (I): 12.1421106447592		92			$P+I\frac{1}{2}+D\frac{N}{2}$	
Derivative (D): 0.0		0.0591928527006381			$s = 1 + N \frac{1}{s}$	
Filter G	Senicienc (N).	2152.865946857.	22		Tune	
Initial c	onditions					
2 million 100 mill						

Figure 8. PID controller parameters after adjustment

If the model created in Simulink is correct, there will be no need for additional adjustments, as this block will automatically adjust (P), (I) and (D) parameters to optimal values.

5. Simulation and results

Simulation and analysis of the results are carried out for the vehicle models presented in Section 3. Values of certain parameters of the vehicle and suspension system are given in Table 1.

Characteristics of vehicles for the masses and the position of center of gravity in Table 1. are obtained by references in [10]. The data for radius of inertia around the x-axis and y are obtained by references in [11]. In the equations, figures (6), (10) and $(11)J_X$ and J_Y are determined according to the literature [11].

$$J_X = m_V \cdot i_X^2 = 409,6 \, kgm^2 \,, \ J_Y = m_V \cdot i_Y^2 = 1277 \, kgm^2 \tag{16}$$

The remaining parameters of the suspension system are obtained from empirical research.

The following section provides diagrams of oscillatory behavior of sprung mass and unsprung mass of the vehicle for all three models. Each diagram shows a comparison between passive and active suspension system. In order to prevent repetition of results and maintain their consistency during presentation of results, only some of the results of oscillatory behavior are shown for each respective model in order to provide a clearer comparison of passive and active suspension systems.

Figures9., 10.and 11. show comparative diagrams of vertical movement, velocity and acceleration of sprung mass for¹/₄ vehicle model for case of passive and active suspension system. Road profile is defined as a trapezoidal obstacle on the road.

It can be seen from Figures9., 10.and 11. that with the active suspension applied to ¹/₄ vehicle model, a decrease in amplitude and period of oscillation occurs.



Figure 9. Vertical travel of sprung mass m_V



Figure 10. Sprung mass m_V vertical travel speed



Figure 11. Sprung mass m_V vertical travel acceleration

Figure 12. shows vertical travel of the wheels (unsprung mass) in two-dimensional oscillatory models in case of passive andactive suspension systems. In this case, the impulse in the system is represented by two obstacles on the road in the form of a square.

It can be seen from Figure 12. that the regulation of suspension using a PID controller reduces the amplitude and period of oscillationsas well as the vertical movement of unsprung mass (wheel) of vehicle.

In order to analyse what happens to the sprung mass during vehicle movement, Figures 13., 14. and 15. show the angle of rotation, angular velocity and angular acceleration of rotation of the sprung mass around the *y*-axis.



Figure 12. Two-dimensional model vertical travel of unprung masses



Figure 13. Rotation angle of sprung $mass(m_V)$ around y-axis



Figure 14. Angular speed of sprung mass (m_V) around y-axis



Figure 15. Angular acceleration of sprung mass (m_V) around y-axis

It can be seen from Figures 13., 14., and 15.that by applying PID controller with this oscillatory model a reduction of the amplitude of oscillation for sprung mass m_V was achieved.

Table $1 7$	Vehicle	characteristics
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Description	Label	Value		
One-dimensionalvehicle model				
Sprung mass	m _V	250 kg		
Unsprung mass	m _T	30 kg		
Suspension system stiffness	CV	25000 N/m		
Tire stiffness	c _T	145000 N/m		
Suspension system damping	k _V	2200 Ns/m		
Two-dimensional vehicle model				
Sprung mass	m _V	500 kg		

Front unsprung mass	m _{TP}	60 kg
Rear unsprung mass	m _{TZ}	60 kg
Front and rear suspension system stiffness	c _{VP} =c _{VZ}	25000 N/m
Front and rear tire stiffness	c _{TP} =c _{TZ}	145000 N/m
Front and rear suspension system damping	k _{VP} =k _{VZ}	1500 Ns/m
Radius of inertia around y-axis	i _Y	1,13 m
Vehicle centre of gravity in longitudinal plane	a ₁	0,933 m
veniere contre of gravity in fongituation plane	a ₂	1,557 m
Three-dimensional vehicle model	·	
Sprung mass	m _V	1000 kg
Front left and right unsprung mass	m _{TPL} =m _{TPD}	30 kg
Rear left and right unsprung mass	m _{TZL} =m _{TZD}	30 kg
Front and rear suspension system stiffness	c _{VP} =c _{VZ}	25000 N/m
Front and rear tire stiffness	c _{TP} =c _{TZ}	145000 N/m
Torsion stabilizer stiffness	c _{TS}	45000 N/m
Front and rear suspension system damping	k _{VP} =k _{VZ}	1500 Ns/m
Radius of inertia around x-axis	i _X	0,64 m
Radius of inertia around y-axis	i _Y	1,13 m
Vehicle centre of gravity in longitudinal plane	a ₁	0,933 m
veniere contre of gravity in fongituation plane	a ₂	1,557 m
Vehicle center of gravity in transverse plane	b ₁	0,706 m
veniere center of gravity in transverse plane	b ₂	0,706 m

Results of the simulations of passive and active oscillatory model of a full vehicle are provided next. Road profile which the vehicle is moving on is defined in Figure 16.

Analysis of the change of angle rotations of sprung mass around *x*-axis (angle \Box) and aroundy-axis (angle \Box) was performed. Results were presented in Figures 17. and 18.



Figure 16. Road profile to be used in three-dimensional model analysis



Figure 17. Rotation angle of sprung mass (m_V) around x-axis



Figure 18. Rotation angle of sprung mass (m_V) around y-axis

Figures 17. and 18. show that by applying PID controller with oscillatory model of full vehicle the amplitude of oscillation of sprung mass around*x* and*y* axis of the analysed model was decreased, and settling time is faster, too.

6. Conclusion

Analysis of oscillatory models of vehicles carried out in this paper has shown that the mathematical models that describe the oscillatory passive suspension systems can be easily modified, so as to describe the models with active suspension systems. Three oscillatory models, described herein, have included PID controller as an element of regulation which is installed between sprung mass and unsprung mass.

It is shown that by using block of PID controller in Simulink program automatic adjustment of parameters of PID controller can be performed. Thus avoiding the complicated setting the parameters of the PID controller by methods available in the literature. Automatically set values of parameters of PID controller have shown in the analysis of oscillatory behavior of certain parameters in oscillating models the justification of use of this type of controller for modeling active suspension system of vehicles.

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Nomenclature

Label	Unit	Description
a ₁ , a ₂	[m]	Coordinates of vehicle gravity center in vehicle longitudinal plane
b ₁ , b ₂	[m]	Coordinates of vehicle gravity center in vehicle transverse plane
c _T	[N/m]	Tire stiffness
c _{TP}	[N/m]	Front tire stiffness
c _{TS}	[N/m]	Torsion stabilizer stiffness
c_{TZ}	[N/m]	Rear tire stiffness
c_V	[N/m]	Suspension system stiffness
c_{VP}	[N/m]	Front suspension system stiffness
c_{VZ}	[N/m]	Rear suspension system stiffness
f	[N]	Active force in suspension system regulation

$\mathbf{f}_{\mathbf{P}}$	[N]	Active force in front suspension system regulation
f_{PD}	[N]	Active force in front right suspension system regulation
$f_{PL} \\$	[N]	Active force in front left suspension system regulation
F_{x}	[N]	Sum of all forces acting on a vehicle in x-axis direction
F_y	[N]	Sum of all forces acting on a vehicle in y-axis direction
F_Z	[N]	Sum of all forces acting on a vehicle in z-axis direction
$\mathbf{f}_{\mathbf{Z}}$	[N]	Active force in rear suspension system regulation
f_{ZD}	[N]	Active force in rear right suspension system regulation
$f_{ZL} \\$	[N]	Active force in rear left suspension system regulation
i_X	[m]	Radius of inertia around x-axis
i_{Y}	[m]	Radius of inertia around y-axis
$J_{\rm X}$	[kgm ²]	Moment of inertia of sprung mass around x-axis
J_{Y}	[kgm ²]	Moment of inertia of sprung mass around y-axis
K _p		Proportional coefficient of PID regulator
$\mathbf{k}_{\mathbf{V}}$	[Ns/m]	Suspension system damping
k_{VP}	[Ns/m]	Front suspension system damping
k_{VZ}	[Ns/m]	Rear suspension system damping
m_{T}	[kg]	Mass of unsprung mass
m _{TP}	[kg]	Mass of front unsprung mass
m _{TZ}	[kg]	Mass of rear unsprung mass
m_V	[kg]	Mass of sprung mass
$M_{\rm X}$	[Nm]	Sum of all moments acting on a vehicle around x-axis
$M_{\rm Y}$	[Nm]	Sum of all moments acting on a vehicle around y-axis
M_{Z}	[Nm]	Sum of all moments acting on a vehicle around z-axis
T_d		Derivative coefficient of PID regulator
T_i		Integral coefficient of PID regulator
ZP	[m]	Impulse from the road
Z _{PP}	[m]	Impulse from the road acting on a vehicle front axle
Z _{PPD}	[m]	Impulse from the road acting on a vehicle front right wheel
Z _{PPL}	[m]	Impulse from the road acting on a vehicle front left wheel
Z _{PZ}	[m]	Impulse from the road acting on a vehicle rear axle

Z _{PZD}	[m]	Impulse from the road acting on a vehicle rear right wheel
Z _{PZL}	[m]	Impulse from the road acting on a vehicle rear left wheel
ZT	[m]	Travel of unsprung mass in z-axis direction
Z _{TP}	[m]	Travel of front unsprung mass in z-axis direction
Z _{TPD}	[m]	Travel of front right unsprung mass in z-axis diretion
Z _{TPL}	[m]	Travel of front left unsprung mass in z-axis diretion
z _{TZ}	[m]	Travel of rear unsprung mass in z-axis direction
z _{TZD}	[m]	Travel of rear right unsprung mass in z-axis diretion
Z _{TZL}	[m]	Travel of rear left unsprung mass in z-axis diretion
$Z_{\rm V}$	[m]	Travel of sprung mass in z-axis direction
	[rad]	Angle of rotation of sprung mass around x-axis
	[rad]	Angle of rotation of sprung mass around y-axis
	[rad]	Angle of rotation of sprung mass around z-axis